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The SNO Solar Neutrino Data, Neutrinoless Double Beta-Decay and Neutrino Mass Spectrum

S. Pascoli and S. T. Petcov ¹

Scuola Internazionale Superiore di Studi Avanzati, I-34014 Trieste, Italy

Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, I-34014 Trieste, Italy

Abstract

Assuming 3- ν mixing and massive Majorana neutrinos, we analyze the implications of the results of the solar neutrino experiments, including the latest SNO data, which favor the LMA MSW solution of the solar neutrino problem with $\tan^2 \theta_\odot < 1$, for the predictions of the effective Majorana mass $|\langle m \rangle|$ in neutrinoless double beta-decay. Neutrino mass spectra with normal mass hierarchy, with inverted hierarchy and of quasi-degenerate type are considered. For $\cos 2\theta_\odot \gtrsim 0.26$, which follows (at 99.73% C.L.) from the SNO analysis of the solar neutrino data, we find significant lower limits on $|\langle m \rangle|$ in the cases of quasi-degenerate and inverted hierarchy neutrino mass spectrum, $|\langle m \rangle| \gtrsim 0.035$ eV and $|\langle m \rangle| \gtrsim 8.5 \times 10^{-3}$ eV, respectively. If the spectrum is hierarchical the upper limit holds $|\langle m \rangle| \lesssim 8.2 \times 10^{-3}$ eV. Correspondingly, not only a measured value of $|\langle m \rangle| \neq 0$, but even an experimental upper limit on $|\langle m \rangle|$ of the order of $\text{few} \times 10^{-2}$ eV can provide information on the type of the neutrino mass spectrum; it can provide also a significant upper limit on the mass of the lightest neutrino m_1 . A measured value of $|\langle m \rangle| \gtrsim 0.2$ eV, combined with data on neutrino masses from the ^3H β -decay experiment KATRIN, might allow to establish whether the CP-symmetry is violated in the lepton sector.

¹Also at: Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 1784 Sofia, Bulgaria

1 Introduction

With the publication of the new results of the SNO solar neutrino experiment [1, 2] (see also [3]) on i) the measured rates of the charged current (CC) and neutral current (NC) reactions, $\nu_e + D \rightarrow e^- + p + p$ and $\nu_l (\bar{\nu}_l) + D \rightarrow \nu_l (\bar{\nu}_l) + n + p$, ii) on the day-night (D-N) asymmetries in the CC and NC reaction rates, and iii) on the day and night event energy spectra, further strong evidences for oscillations or transitions of the solar ν_e into active neutrinos $\nu_{\mu,\tau}$ (and/or antineutrinos $\bar{\nu}_{\mu,\tau}$), taking place when the solar ν_e travel from the central region of the Sun to the Earth, have been obtained. The evidences for oscillations (or transitions) of the solar ν_e become even stronger when the SNO data are combined with the data obtained in the other solar neutrino experiments, Homestake, Kamiokande, SAGE, GALLEX/GNO and Super-Kamiokande [4, 5].

Global analysis of the solar neutrino data [1, 2, 3, 4, 5], including the latest SNO results, in terms of the hypothesis of oscillations of the solar ν_e into active neutrinos, $\nu_e \rightarrow \nu_{\mu(\tau)}$, show [1] that the data favor the large mixing angle (LMA) MSW solution with $\tan^2 \theta_\odot < 1$, where θ_\odot is the angle which controls the solar neutrino transitions. The LOW solution of the solar neutrino problem with transitions into active neutrinos is only allowed at approximately 99.73% C.L. [1]; there do not exist other solutions at the indicated confidence level. In the case of the LMA solution, the range of values of the neutrino mass-squared difference $\Delta m_\odot^2 > 0$, characterizing the solar neutrino transitions, found in [1] at 99.73% C.L. reads:

$$\text{LMA MSW :} \quad 2.2 \times 10^{-5} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 2.0 \times 10^{-4} \text{ eV}^2 \quad (99.73\% \text{ C.L.}). \quad (1)$$

The best fit value of Δm_\odot^2 obtained in [1] is $(\Delta m_\odot^2)_{\text{BF}} = 5.0 \times 10^{-5} \text{ eV}^2$. The mixing angle θ_\odot was found in the case of the LMA solution to lie in an interval which at 99.73% C.L. is determined by [1]

$$\text{LMA MSW :} \quad 0.26 \lesssim \cos 2\theta_\odot \lesssim 0.64 \quad (99.73\% \text{ C.L.}). \quad (2)$$

The best fit value of $\cos 2\theta_\odot$ in the LMA solution region is given by $(\cos 2\theta_\odot)_{\text{BF}} = 0.50$.

Strong evidences for oscillations of atmospheric neutrinos have been obtained in the Super-Kamiokande experiment [6]. As is well known, the atmospheric neutrino data is best described in terms of dominant $\nu_\mu \rightarrow \nu_\tau$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$) oscillations. The explanation of the solar and atmospheric neutrino data in terms of neutrino oscillations requires the existence of 3-neutrino mixing in the weak charged lepton current (see, e.g., [7, 8]).

Assuming 3- ν mixing and massive Majorana neutrinos, we analyze the implications of the latest results of the SNO experiment for the predictions of the effective Majorana mass $|<m>|$ in neutrinoless double beta $((\beta\beta)_{0\nu^-})$ decay (see, e.g., [9, 10, 11]):

$$|<m>| = |m_1|U_{e1}|^2 + m_2|U_{e2}|^2 e^{i\alpha_{21}} + m_3|U_{e3}|^2 e^{i\alpha_{31}}|. \quad (3)$$

Here $m_{1,2,3}$ are the masses of 3 Majorana neutrinos with definite mass $\nu_{1,2,3}$, U_{ej} are elements of the lepton mixing matrix U - the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [12, 13], and α_{21} and α_{31} are two Majorana CP-violating phases² [14, 15]. If CP-invariance holds, one has [16, 17] $\alpha_{21} = k\pi$, $\alpha_{31} = k'\pi$, $k, k' = 0, 1, 2, \dots$, and

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1, \quad (4)$$

represent the relative CP-parities of the neutrinos ν_1 and ν_2 , and ν_1 and ν_3 , respectively.

The experiments searching for $(\beta\beta)_{0\nu^-}$ -decay test the underlying symmetries of particle interactions (see, e.g., [9]). They can answer the fundamental question about the nature of massive

²We assume that the fields of the Majorana neutrinos ν_j satisfy the Majorana condition: $C(\bar{\nu}_j)^T = \nu_j$, $j = 1, 2, 3$, where C is the charge conjugation matrix.

neutrinos, which can be Dirac or Majorana fermions. If the massive neutrinos are Majorana particles, the observation of $(\beta\beta)_{0\nu}$ -decay³ can provide unique information on the type of the neutrino mass spectrum and on the lightest neutrino mass [10, 11, 21, 22, 23, 24, 25, 26]. Combined with data from the ^3H β -decay neutrino mass experiment KATRIN [27], it can give also unique information on the CP-violation in the lepton sector induced by the Majorana CP-violating phases, and if CP-invariance holds - on the relative CP-parities of the massive Majorana neutrinos [10, 11, 22, 28].

Rather stringent upper bounds on $|\langle m \rangle|$ have been obtained in the ^{76}Ge experiments by the Heidelberg-Moscow collaboration [29], $|\langle m \rangle| < 0.35$ eV (90%C.L.), and by the IGEX collaboration [30], $|\langle m \rangle| < (0.33 \div 1.35)$ eV (90%C.L.). Taking into account a factor of 3 uncertainty in the calculated value of the corresponding nuclear matrix element, we get for the upper limit found in [29]: $|\langle m \rangle| < 1.05$ eV. Considerably higher sensitivity to the value of $|\langle m \rangle|$ is planned to be reached in several $(\beta\beta)_{0\nu}$ -decay experiments of a new generation. The NEMO3 experiment [31], which will begin to take data in July of 2002, and the cryogenic detector CUORE [32], are expected to reach a sensitivity to values of $|\langle m \rangle| \cong 0.1$ eV. An order of magnitude better sensitivity, i.e., to $|\langle m \rangle| \cong 10^{-2}$ eV, is planned to be achieved in the GENIUS experiment [33] utilizing one ton of enriched ^{76}Ge , and in the EXO experiment [34], which will search for $(\beta\beta)_{0\nu}$ -decay of ^{136}Xe . Two more detectors, Majorana [35] and MOON [36], are planned to have sensitivity to $|\langle m \rangle|$ in the range of $few \times 10^{-2}$ eV.

In what regards the ^3H β -decay experiments, the currently existing most stringent upper bounds on the electron (anti-)neutrino mass $m_{\bar{\nu}_e}$ were obtained in the Troitzk [37] and Mainz [38] experiments and read $m_{\bar{\nu}_e} < 2.2$ eV. The KATRIN ^3H β -decay experiment [27] is planned to reach a sensitivity to $m_{\bar{\nu}_e} \sim 0.35$ eV.

The fact that the solar neutrino data implies a relatively large lower limit on the value of $\cos 2\theta_\odot$, eq. (2), has important implications for the predictions of the effective Majorana mass parameter in $(\beta\beta)_{0\nu}$ -decay [10, 11] and in the present article we investigate these implications.

2 The SNO Data and the Predictions for the Effective Majorana Mass $|\langle m \rangle|$

According to the analysis performed in [1], the solar neutrino data, including the latest SNO results, strongly favor the LMA solution of the solar neutrino problem with $\tan^2 \theta_\odot < 1$. We take into account these new development to update the predictions for the effective Majorana mass $|\langle m \rangle|$, derived in [10], and the analysis of the implications of the measurement of, or obtaining a more stringent upper limit on, $|\langle m \rangle|$ performed in [10, 11]. The predicted value of $|\langle m \rangle|$ depends in the case of 3-neutrino mixing of interest on (see e.g. [10, 11, 25]): i) the value of the lightest neutrino mass m_1 , ii) Δm_\odot^2 and θ_\odot , iii) the neutrino mass-squared difference which characterizes the atmospheric ν_μ ($\bar{\nu}_\mu$) oscillations, Δm_{atm}^2 , and iv) the lepton mixing angle θ which is limited by the CHOOZ and Palo Verde experiments [39, 40]. The ranges of allowed values of Δm_\odot^2 and θ_\odot are determined in [1], while those of Δm_{atm}^2 and θ are taken from [41] (we use the best fit values and the 99% C.L. results from [41]). Given the indicated parameters, the value of $|\langle m \rangle|$ depends strongly [10, 11] on the type of the neutrino mass spectrum, as well as on the values of the two Majorana CP-violating phases, α_{21} and α_{31} (see eq. (3)), present in the lepton mixing matrix.

Let us note that if Δm_{atm}^2 lies in the interval $\Delta m_{\text{atm}}^2 \cong (2.0 - 5.0) \times 10^{-3}$ eV², as is suggested by the current atmospheric neutrino data [6], its value will be determined with a high precision ($\sim 10\%$ uncertainty) by the MINOS experiment [42]. Similarly, if $\Delta m_\odot^2 \cong (2.5 - 10.0) \times 10^{-5}$ eV², which is favored by the solar neutrino data, the KamLAND experiment will be able to measure Δm_\odot^2 with an uncertainty of $\sim (10 - 15)\%$ (99.73% C.L.). Combining the data from the solar neutrino

³Evidences for $(\beta\beta)_{0\nu}$ -decay taking place with a rate corresponding to $0.11 \text{ eV} \leq |\langle m \rangle| \leq 0.56 \text{ eV}$ (95% C.L.) are claimed to have been obtained in [18]. The results announced in [18] have been criticized in [19, 20].

experiments and from KamLAND would permit to determine $\cos 2\theta_\odot$ with a high precision as well ($\sim 15\%$ uncertainty at 99.73% C.L., see ⁴, e.g., [44]). Somewhat better limits on $\sin^2 \theta$ than the existing one can be obtained in the MINOS experiment.

We number the massive neutrinos (without loss of generality) in such a way that $m_1 < m_2 < m_3$. In the analysis which follows we consider neutrino mass spectra with normal mass hierarchy, with inverted hierarchy and of quasi-degenerate type [10, 11, 21, 22, 23, 24, 26]. In the case of neutrino mass spectrum with normal mass hierarchy ($m_1 \ll (<) m_2 \ll m_3$) we have $\Delta m_\odot^2 \equiv \Delta m_{21}^2$ and $\sin^2 \theta \equiv |U_{e3}|^2$, while in the case of spectrum with inverted hierarchy ($m_1 \ll m_2 \cong m_3$) one finds $\Delta m_\odot^2 \equiv \Delta m_{32}^2$ and $\sin^2 \theta \equiv |U_{e1}|^2$. In both cases one can choose $\Delta m_{\text{atm}}^2 \equiv \Delta m_{31}^2$. It should be noted that for $m_1 > 0.2 \text{ eV} \gg \sqrt{\Delta m_{\text{atm}}^2}$, the neutrino mass spectrum is of the quasi-degenerate type, $m_1 \cong m_2 \cong m_3$, and the two possibilities, $\Delta m_\odot^2 \equiv \Delta m_{21}^2$ and $\Delta m_\odot^2 \equiv \Delta m_{32}^2$, lead to the same predictions for $|\langle m \rangle|$.

2.1 Normal Mass Hierarchy: $\Delta m_\odot^2 \equiv \Delta m_{21}^2$

If $\Delta m_\odot^2 = \Delta m_{21}^2$, the effective Majorana mass parameter $|\langle m \rangle|$ is given in terms of the oscillation parameters Δm_\odot^2 , Δm_{atm}^2 , θ_\odot and $|U_{e3}|^2$ which is constrained by the CHOOZ data, as follows [10]:

$$|\langle m \rangle| = \left| (m_1 \cos^2 \theta_\odot + \sqrt{m_1^2 + \Delta m_\odot^2} \sin^2 \theta_\odot e^{i\alpha_{21}})(1 - |U_{e3}|^2) + \sqrt{m_1^2 + \Delta m_{\text{atm}}^2} |U_{e3}|^2 e^{i\alpha_{31}} \right|. \quad (5)$$

The effective Majorana mass $|\langle m \rangle|$ can lie anywhere between 0 and the present upper limits, as Fig. 1 (left panels) shows ⁵. This conclusion does not change even under the most favorable conditions for the determination of $|\langle m \rangle|$, namely, even when Δm_{atm}^2 , Δm_\odot^2 , θ_\odot and θ are known with negligible uncertainty [11]. Our further conclusions for the case of the LMA solution of the solar neutrino problem [1] are illustrated in Fig. 1 (left panels) and are summarized below.

Case A: $m_1 < 0.02 \text{ eV}$, $m_1 < m_2 \ll m_3$.

Taking into account the new constraints on the solar neutrino oscillation parameters following from the SNO data [1] does not change qualitatively the conclusions reached in ref. [10, 11]. The maximal value of $|\langle m \rangle|$, $|\langle m \rangle|_{\text{MAX}}$, for given m_1 reads:

$$|\langle m \rangle|_{\text{MAX}} = \left(m_1 (\cos^2 \theta_\odot)_{\text{MIN}} + \sqrt{m_1^2 + (\Delta m_\odot^2)_{\text{MAX}}} (\sin^2 \theta_\odot)_{\text{MAX}} \right) (1 - |U_{e3}|_{\text{MAX}}^2) + \sqrt{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MAX}}} |U_{e3}|_{\text{MAX}}^2, \quad (6)$$

where $(\cos^2 \theta_\odot)_{\text{MIN}}$ and $(\sin^2 \theta_\odot)_{\text{MAX}}$ are the values corresponding to $(\tan^2 \theta_\odot)_{\text{MAX}}$, and $(\Delta m_{\text{atm}}^2)_{\text{MAX}}$ is the maximal value of Δm_{atm}^2 allowed for the $|U_{e3}|_{\text{MAX}}^2$ [41].

For the values of Δm_\odot^2 and $\tan^2 \theta_\odot$ from the LMA solution region [1], eqs. (1) and (2), we get for $m_1 \ll 0.02 \text{ eV}$: $|\langle m \rangle|_{\text{MAX}} \simeq 8.2 \times 10^{-3} \text{ eV}$. Using the best fit values of the oscillation parameters found in refs. [1, 41], one obtains: $|\langle m \rangle|_{\text{MAX}} \simeq 2.0 \times 10^{-3} \text{ eV}$. The maximal value of $|\langle m \rangle|$ corresponds to the case of CP-conservation and $\nu_{1,2,3}$ having identical CP-parities, $\eta_{21} = \eta_{31} = 1$.

There is no significant lower bound on $|\langle m \rangle|$ because of the possibility of mutual compensations between the terms contributing to $|\langle m \rangle|$ and corresponding to the exchange of different virtual massive Majorana neutrinos. Furthermore, the uncertainties in the oscillation parameters do not allow to identify a “just-CP violation” region of values of $|\langle m \rangle|$ [10] (a value of $|\langle m \rangle|$ in

⁴We thank C. Peña-Garay for clarifications on this point.

⁵This statement is valid as long as m_1 and the CP violating phases which enter the effective Majorana mass $|\langle m \rangle|$ are not constrained.

this region would unambiguously signal the existence of CP-violation in the lepton sector, caused by Majorana CP-violating phases). However, if the neutrinoless double beta-decay will be observed, the measured value of $|\langle m \rangle|$, combined with information on m_1 and a better determination of the relevant neutrino oscillation parameters, might allow to determine whether the CP-symmetry is violated due to Majorana CP-violating phases, or to identify which are the allowed patterns of the massive neutrino CP-parities in the case of CP-conservation (for a detailed discussion see ref. [11]).

Case B: Neutrino Mass Spectrum with Partial Hierarchy ($0.02 \text{ eV} \leq m_1 \leq 0.2 \text{ eV}$)

For $m_1 \geq 0.02 \text{ eV}$ there exists a lower bound on the possible values of $|\langle m \rangle|$ (Fig. 1, left panels). Using the 99.73% C.L. allowed values of Δm_\odot^2 and $\cos 2\theta_\odot$ from [1], we find that this lower bound is significant, i.e., $|\langle m \rangle| \gtrsim 10^{-2} \text{ eV}$, for $m_1 \gtrsim 0.07 \text{ eV}$. For the best fit values of the oscillation parameters obtained in [1, 41], one has $|\langle m \rangle| \gtrsim 10^{-2} \text{ eV}$ for $m_1 \geq 0.02 \text{ eV}$.

For a fixed $m_1 \geq 0.02 \text{ eV}$, the minimal value of $|\langle m \rangle|$, $|\langle m \rangle|_{\text{MIN}}$, is given by

$$|\langle m \rangle|_{\text{MIN}} \cong m_1 (\cos 2\theta_\odot)_{\text{MIN}} (1 - |U_{e3}|_{\text{MAX}}^2) - \sqrt{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MAX}}} |U_{e3}|_{\text{MAX}}^2 + \mathcal{O}\left(\frac{(\Delta m_\odot^2)_{\text{MAX}}}{4m_1}\right), \quad (7)$$

where again $(\Delta m_{\text{atm}}^2)_{\text{MAX}}$ is the maximal allowed value of Δm_{atm}^2 for the $|U_{e3}|_{\text{MAX}}^2$ [41]. The upper bound on $|\langle m \rangle|$, which corresponds to CP-conservation and $\eta_{21} = \eta_{31} = +1$ ($\nu_{1,2,3}$ possessing identical CP-parities), can be found for given m_1 by using eq. (6). For the allowed values of m_1 , $0.02 \text{ eV} \leq m_1 \leq 0.2 \text{ eV}$, we have $|\langle m \rangle| \leq 0.2 \text{ eV}$.

2.2 Inverted Neutrino Mass Hierarchy: $\Delta m_\odot^2 \equiv \Delta m_{32}^2$

If $\Delta m_\odot^2 = \Delta m_{32}^2$, the effective Majorana mass $|\langle m \rangle|$ is given in terms of the oscillation parameters Δm_\odot^2 , Δm_{atm}^2 , θ_\odot and $|U_{e1}|^2$ which is constrained by the CHOOZ data [10]:

$$|\langle m \rangle| = \left| m_1 |U_{e1}|^2 + \sqrt{m_1^2 + \Delta m_{\text{atm}}^2 - \Delta m_\odot^2} \cos^2 \theta_\odot (1 - |U_{e1}|^2) e^{i\alpha_{21}} + \sqrt{m_1^2 + \Delta m_{\text{atm}}^2} \sin^2 \theta_\odot (1 - |U_{e1}|^2) e^{i\alpha_{31}} \right|. \quad (8)$$

The new predictions for $|\langle m \rangle|$ differ substantially from those obtained before the appearance of the latest SNO data due to the existence of a significant lower bound on $|\langle m \rangle|$ for every value of m_1 : even in the case of $m_1 \ll m_2 \cong m_3$ (i.e., even if $m_1 \ll 0.02 \text{ eV}$), we get

$$|\langle m \rangle| \gtrsim 8.5 \times 10^{-3} \text{ eV} \quad (9)$$

(see Fig. 1, right panels). Given the neutrino oscillation parameters, the minimal allowed value of $|\langle m \rangle|$ depends on the values of the CP violating phases α_{21} and α_{31} .

Case A: $m_1 < 0.02 \text{ eV}$, $m_1 \ll m_2 \simeq m_3$.

The effective Majorana mass $|\langle m \rangle|$ can be considerably larger than in the case of a hierarchical neutrino mass spectrum [10, 23]. The maximal value of $|\langle m \rangle|$ corresponds to CP-conservation and $\eta_{21} = \eta_{31} = +1$, and for given m_1 reads:

$$|\langle m \rangle|_{\text{MAX}} = m_1 |U_{e1}|_{\text{MIN}}^2 + \left(\sqrt{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MAX}} - (\Delta m_\odot^2)_{\text{MIN}}} (\cos^2 \theta_\odot)_{\text{MIN}} + \sqrt{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MAX}}} (\sin^2 \theta_\odot)_{\text{MAX}} \right) (1 - |U_{e1}|_{\text{MIN}}^2), \quad (10)$$

where $(\cos^2 \theta_\odot)_{\text{MIN}}$ and $(\sin^2 \theta_\odot)_{\text{MAX}}$ are the values corresponding to $(\tan^2 \theta_\odot)_{\text{MAX}}$, and $|U_{e1}|_{\text{MIN}}^2$ is the minimal allowed value of $|U_{e1}|^2$ for the $(\Delta m_{\text{atm}}^2)_{\text{MAX}}$. For the allowed ranges - eqs. (1) and (2) for Δm_\odot^2 and $\tan^2 \theta_\odot$, and the best fit values of the neutrino oscillation parameters, found in [1, 41], we get $|\langle m \rangle|_{\text{MAX}} \simeq 0.080 \text{ eV}$ and $|\langle m \rangle|_{\text{MAX}} \simeq 0.056 \text{ eV}$, respectively.

There exists a non-trivial lower bound on $|\langle m \rangle|$ in the case of the LMA solution for which $\cos 2\theta_\odot$ is found to be significantly different from zero. For the 99.73% C.L. allowed values of Δm_\odot^2 and $\cos 2\theta_\odot$ [1], this lower bound reads: $|\langle m \rangle| \gtrsim 8.5 \times 10^{-3}$ eV. Using the best fit values of the oscillation parameters [1, 41], we find:

$$|\langle m \rangle| \gtrsim 2.8 \times 10^{-2} \text{ eV}. \quad (11)$$

The lower bound is present even for $\cos 2\theta_\odot > 0.1$: in this case $|\langle m \rangle| \gtrsim 4 \times 10^{-3}$ eV. The minimal value of $|\langle m \rangle|$, $|\langle m \rangle|_{\text{MIN}}$, is reached in the case of CP-invariance and $\eta_{21} = -\eta_{31} = -1$, and is determined by:

$$|\langle m \rangle|_{\text{MIN}} = \left| m_1 |U_{e1}|_{\text{MAX}}^2 - \left(\sqrt{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MIN}} - (\Delta m_\odot^2)_{\text{MAX}}} (\cos^2 \theta_\odot)_{\text{MIN}} - \sqrt{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MIN}}} (\sin^2 \theta_\odot)_{\text{MAX}} \right) (1 - |U_{e1}|_{\text{MAX}}^2) \right|, \quad (12)$$

where $(\cos^2 \theta_\odot)_{\text{MIN}}$ and $(\sin^2 \theta_\odot)_{\text{MAX}}$ are the values corresponding to $(\tan^2 \theta_\odot)_{\text{MAX}}$, and $|U_{e1}|_{\text{MAX}}^2$ is the maximal allowed value of $|U_{e1}|^2$ for the $(\Delta m_{\text{atm}}^2)_{\text{MIN}}$. In the two other CP conserving cases of $\eta_{21} = \eta_{31} = \pm 1$, the lower bound on $|\langle m \rangle|$ depends weakly on the allowed values of θ_\odot and reads $|\langle m \rangle| \gtrsim 0.03$ eV.

If the neutrino mass spectrum is of the inverted hierarchy type, a sufficiently precise determination of Δm_{atm}^2 , θ_\odot and $|U_{e1}|^2$ (or a better upper limit on $|U_{e1}|^2$), combined with a measurement of $|\langle m \rangle|$ in the current or future $(\beta\beta)_{0\nu}$ -decay experiments, could allow one to get information on the difference of the Majorana CP-violating phases $(\alpha_{31} - \alpha_{21})$ [22]. The value of $\sin^2(\alpha_{31} - \alpha_{21})/2$ is related to the experimentally measurable quantities as follows [10, 22]:

$$\sin^2 \frac{\alpha_{31} - \alpha_{21}}{2} \simeq \left(1 - \frac{|\langle m \rangle|^2}{(m_1^2 + \Delta m_{\text{atm}}^2)(1 - |U_{e1}|^2)^2} \right) \frac{1}{\sin^2 2\theta_\odot} \simeq \left(1 - \frac{|\langle m \rangle|^2}{\Delta m_{\text{atm}}^2 (1 - |U_{e1}|^2)^2} \right) \frac{1}{\sin^2 2\theta_\odot}, \quad (13)$$

($m_1 < 0.02$ eV). The constraints on $\sin^2(\alpha_{31} - \alpha_{21})/2$ one could derive on the basis of eq. (13) are illustrated ⁶ in Fig. 11 of ref. [10]. Obtaining an experimental upper limit on $|\langle m \rangle|$ of the order of 0.03 eV would permit, in particular, to get a lower bound on the value of $\sin^2(\alpha_{31} - \alpha_{21})/2$ and possibly exclude the CP conserving case corresponding to $\alpha_{31} - \alpha_{21} = 0$ (i.e., $\eta_{21} = \eta_{31} = \pm 1$).

Case B: Spectrum with Partial Inverted Hierarchy ($0.02 \text{ eV} \leq m_1 \leq 0.2 \text{ eV}$).

The discussion and conclusions in the case of the spectrum with partial inverted hierarchy are identical to those in the same case for the neutrino mass spectrum with normal hierarchy given in sub-section 2.1, Case B, except for the maximal and minimal values of $|\langle m \rangle|$, $|\langle m \rangle|_{\text{MAX}}$ and $|\langle m \rangle|_{\text{MIN}}$, which for a fixed m_1 are determined by:

$$|\langle m \rangle|_{\text{MAX}} \simeq m_1 |U_{e1}|_{\text{MIN}}^2 + \sqrt{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MAX}}} (1 - |U_{e1}|_{\text{MIN}}^2), \quad (14)$$

$$|\langle m \rangle|_{\text{MIN}} \simeq \left| m_1 |U_{e1}|_{\text{MAX}}^2 - \sqrt{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MIN}}} (\cos 2\theta_\odot)_{\text{MIN}} (1 - |U_{e1}|_{\text{MAX}}^2) \right|, \quad (15)$$

$|U_{e1}|_{\text{MIN}}^2$ ($|U_{e1}|_{\text{MAX}}^2$) in eq. (14) (in eq. (15)) being the minimal (maximal) allowed value of $|U_{e1}|^2$ given the maximal (minimal) value $(\Delta m_{\text{atm}}^2)_{\text{MAX}}$ ($(\Delta m_{\text{atm}}^2)_{\text{MIN}}$).

For any $m_1 \geq 0.02$ eV, the lower bound on $|\langle m \rangle|$ reads: $|\langle m \rangle| \gtrsim 0.01$ eV. Using the best fit values of the neutrino oscillation parameters, obtained in [1, 41], one finds: $|\langle m \rangle| \gtrsim 0.03$ eV.

⁶Note that the CP-violating phase α_{21} is not constrained in the case under discussion. Even if it is found that $\alpha_{31} - \alpha_{21} = 0, \pm\pi$, α_{21} can be a source of CP-violation in $\Delta L = 2$ processes other than $(\beta\beta)_{0\nu}$ -decay.

2.3 Quasi-Degenerate Mass Spectrum ($m_1 > 0.2$ eV, $m_1 \simeq m_2 \simeq m_3 \simeq m_{\bar{\nu}_e}$)

The new element in the predictions for $|\langle m \rangle|$ in the case of quasi-degenerate neutrino mass spectrum, $m_1 > 0.2$ eV, is the existence of a lower bound on the possible values of $|\langle m \rangle|$ (Fig. 1). The lower limit on $|\langle m \rangle|$ is reached in the case of CP-conservation and $\eta_{21} = \eta_{31} = -1$. One finds a significant lower limit, $|\langle m \rangle| \gtrsim 0.01$ eV, if

$$(\cos 2\theta_\odot)_{\text{MIN}} > \max(0.05, 1.5 |U_{e3}|_{\text{MAX}}^2 / (1 - |U_{e3}|_{\text{MAX}}^2)). \quad (16)$$

More specifically, using the best fit value, and the 90% C.L. and the 99.73% C.L. allowed values, of $\cos 2\theta_\odot$ from [1], we obtain, respectively: $|\langle m \rangle| \geq 0.10$ eV, $|\langle m \rangle| \geq 0.06$ eV and

$$|\langle m \rangle| \geq 0.035 \text{ eV} \quad (17)$$

(Figs. 1 and 3). These values of $|\langle m \rangle|$ are in the range of sensitivity of the current and future $(\beta\beta)_{0\nu}$ -decay experiments.

The upper bound on $|\langle m \rangle|$, which corresponds to CP-conservation and $\eta_{21} = \eta_{31} = +1$ ($\nu_{1,2,3}$ possessing the same CP-parities), can be found for a given m_1 by using eq. (6). For the allowed values of $m_1 > 0.2$ eV (which is limited from above by the ^3H β -decay data [37, 38], $m_{1,2,3} \simeq m_{\bar{\nu}_e}$), $|\langle m \rangle|_{\text{MAX}}$ is limited by the upper bounds obtained in the $(\beta\beta)_{0\nu}$ -decay experiments: $|\langle m \rangle| < 0.35$ eV [29] and $|\langle m \rangle| < (0.33 - 1.35)$ eV [30].

In the case of CP conservation and $\eta_{21} = \pm\eta_{31} = +1$, $|\langle m \rangle|$ is constrained to lie in the interval $[10] \ m_{\bar{\nu}_e}(1 - 2|U_{e3}|_{\text{MAX}}^2) \leq |\langle m \rangle| \leq m_{\bar{\nu}_e}$. An upper limit on $|\langle m \rangle|$ would lead to an upper limit on $m_{\bar{\nu}_e}$ which is more stringent than the one obtained in the present ^3H β -decay experiments: for $|\langle m \rangle| < 0.35$ (1.05) eV we have $m_{\bar{\nu}_e} < 0.41$ (1.23) eV. Furthermore, the upper limit $|\langle m \rangle| < 0.2$ eV would permit to exclude the CP-parity pattern $\eta_{21} = \pm\eta_{31} = +1$ for the quasi-degenerate neutrino mass spectrum.

If the CP-symmetry holds and $\eta_{21} = \pm\eta_{31} = -1$, there are both an upper and a lower limits on $|\langle m \rangle|$, $m_{\bar{\nu}_e}((\cos 2\theta_\odot)_{\text{MIN}}(1 - |U_{e3}|_{\text{MIN}}^2) + |U_{e3}|_{\text{MIN}}^2) \leq |\langle m \rangle| \leq m_{\bar{\nu}_e}((\cos 2\theta_\odot)_{\text{MAX}}(1 - |U_{e3}|_{\text{MAX}}^2) + |U_{e3}|_{\text{MAX}}^2)$. Using eq. (2) and the results on $|U_{e3}|^2$ from ref. [41], one finds $0.26 \ m_{\bar{\nu}_e} \leq |\langle m \rangle| \leq 0.67 \ m_{\bar{\nu}_e}$. Given the allowed values of $\cos 2\theta_\odot$, eq. (2), the observation of the $(\beta\beta)_{0\nu}$ -decay in the present and/or future $(\beta\beta)_{0\nu}$ -decay experiments, combined with a sufficiently stringent upper bound on $m_{\bar{\nu}_e} \simeq m_{1,2,3}$ from the tritium beta-decay experiments, $m_{\bar{\nu}_e} < |\langle m \rangle|_{\text{exp}} / ((\cos 2\theta_\odot)_{\text{MAX}}(1 - |U_{e3}|_{\text{MAX}}^2) + |U_{e3}|_{\text{MAX}}^2)$, would allow one, in particular, to exclude the case of CP-conservation with $\eta_{21} = \pm\eta_{31} = -1$ (Fig. 2).

For values of $|\langle m \rangle|$, which are in the range of sensitivity of the future $(\beta\beta)_{0\nu}$ -decay experiments, there exists a “just-CP-violation” region. This is illustrated in Fig. 2, where we show $|\langle m \rangle| / m_1$ for the case of quasi-degenerate neutrino mass spectrum, $m_1 > 0.2$ eV, $m_1 \simeq m_2 \simeq m_3 \simeq m_{\bar{\nu}_e}$, as a function of $\cos 2\theta_\odot$. The “just-CP-violation” interval of values of $|\langle m \rangle| / m_1$ is determined by

$$(\cos 2\theta_\odot)_{\text{MAX}}(1 - |U_{e3}|_{\text{MAX}}^2) + |U_{e3}|_{\text{MAX}}^2 < \frac{|\langle m \rangle|}{m_{\bar{\nu}_e}} < 1 - 2|U_{e3}|_{\text{MAX}}^2. \quad (18)$$

Taking into account eq. (2) and the existing limits on $|U_{e3}|^2$, this gives $0.67 < |\langle m \rangle| / m_{\bar{\nu}_e} < 0.85$. Information about the masses $m_{1,2,3} \simeq m_{\bar{\nu}_e}$ can be obtained in the KATRIN experiment [27].

A rather precise determination of $|\langle m \rangle|$, $m_1 \simeq m_{\bar{\nu}_e}$, θ_\odot and $|U_{e3}|^2$ would imply an inter-dependent constraint on the two CP-violating phases α_{21} and α_{31} [10] (see Fig. 16 in [10]). For $m_1 \equiv m_{\bar{\nu}_e} > 0.2$ eV, the CP-violating phase α_{21} could be tightly constrained if $|U_{e3}|^2$ is sufficiently small and the term in $|\langle m \rangle|$ containing it can be neglected, as is suggested by the current limits on $|U_{e3}|^2$:

$$\sin^2 \frac{\alpha_{21}}{2} \simeq \left(1 - \frac{|\langle m \rangle|^2}{m_{\bar{\nu}_e}^2}\right) \frac{1}{\sin^2 2\theta_\odot}. \quad (19)$$

The term which depends on the CP-violating phase α_{31} in the expression for $|\langle m \rangle|$, is suppressed by the factor $|U_{e3}|^2$. Therefore the constraint one could possibly obtain on $\cos \alpha_{31}$ is trivial (Fig. 16 in [10]), unless $|U_{e3}|^2 \sim \mathcal{O}(\sin^2 \theta_\odot)$.

3 The Effective Majorana Mass and the Determination of the Neutrino Mass Spectrum

The existence of a lower bound on $|\langle m \rangle|$ in the cases of inverted mass hierarchy ($\Delta m_\odot^2 = \Delta m_{32}^2$) and quasi-degenerate neutrino mass spectrum, eqs. (9) and (17), implies that the future $(\beta\beta)_{0\nu}$ -decay experiments might allow to determine the type of the neutrino mass spectrum (under the general assumptions of 3-neutrino mixing and massive Majorana neutrinos, $(\beta\beta)_{0\nu}$ -decay generated only by the (V-A) charged current weak interaction via the exchange of the three Majorana neutrinos, neutrino oscillation explanation of the solar and atmospheric neutrino data). This conclusion is valid not only under the assumption that the $(\beta\beta)_{0\nu}$ -decay will be observed in these experiments and $|\langle m \rangle|$ will be measured, but also in the case only a sufficiently stringent upper limit on $|\langle m \rangle|$ will be derived.

More specifically, as is illustrated in Fig. 3, the following statements can be made:

1. a measurement of $|\langle m \rangle| = |\langle m \rangle|_{exp} > 0.20$ eV, would imply that the neutrino mass spectrum is of the quasi-degenerate type ($m_1 > 0.20$ eV) and that there are both a lower and an upper limit on m_1 , $(m_1)_{min} \leq m_1 \leq (m_1)_{max}$. The values of $(m_1)_{max}$ and $(m_1)_{min}$ are fixed respectively by the equalities $|\langle m \rangle|_{min} = |\langle m \rangle|_{exp}$ and $|\langle m \rangle|_{max} = |\langle m \rangle|_{exp}$, where $|\langle m \rangle|_{min}$ and $|\langle m \rangle|_{max}$ are given by eqs. (7) and (6);
2. if $|\langle m \rangle|$ is measured and is found to lie in the interval 8.5×10^{-2} eV $\lesssim |\langle m \rangle|_{exp} \lesssim 0.20$ eV, one could conclude that either
 - i) $\Delta m_\odot^2 \equiv \Delta m_{21}^2$ and the spectrum is of the quasi-degenerate type ($m_1 > 0.20$ eV) or with partial hierarchy (0.02 eV $\leq m_1 \leq 0.2$ eV), with 8.4×10^{-2} eV $\lesssim m_1 \lesssim 1.2$ eV, where the maximal and minimal values of m_1 are determined as in the *Case 1*;
 - or that ii) $\Delta m_\odot^2 \equiv \Delta m_{32}^2$ and the spectrum is quasi-degenerate ($m_1 > 0.20$ eV) or with partial inverted hierarchy (0.02 eV $\leq m_1 \leq 0.2$ eV), with $(m_1)_{min} = 2.0 \times 10^{-2}$ eV and $(m_1)_{max} = 1.2$ eV, where $(m_1)_{max}$ and $(m_1)_{min}$ are given by the equalities $|\langle m \rangle|_{min} = |\langle m \rangle|_{exp}$ and $|\langle m \rangle|_{max} = |\langle m \rangle|_{exp}$, and $|\langle m \rangle|_{min}$ and $|\langle m \rangle|_{max}$ are determined by eqs. (15) and (14);
3. a measured value of $|\langle m \rangle|$ satisfying 8.5×10^{-3} eV $\lesssim |\langle m \rangle|_{exp} \lesssim 8.0 \times 10^{-2}$ eV, would imply that (see Fig. 3) either
 - i) $\Delta m_\odot^2 \equiv \Delta m_{21}^2$ and the spectrum is of quasi-degenerate type ($m_1 > 0.20$ eV), with $(m_1)_{max} \lesssim 0.48$ eV, or with partial hierarchy (0.02 eV $\leq m_1 \leq 0.2$ eV),
 - or that ii) $\Delta m_\odot^2 \equiv \Delta m_{32}^2$ and the spectrum is quasi-degenerate ($m_1 > 0.20$ eV), or with partial inverted hierarchy (0.02 eV $\leq m_1 \leq 0.2$ eV), or with inverted hierarchy ($m_1 < 0.02$ eV), with only a significant upper bound on m_1 , $(m_1)_{min} = 0$, $(m_1)_{max} \lesssim 0.48$ eV, where $(m_1)_{max}$ is determined by the equation $|\langle m \rangle|_{min} = |\langle m \rangle|_{exp}$, with $|\langle m \rangle|_{min}$ given by eq. (15);
4. a measurement or an upper limit on $|\langle m \rangle|$, $|\langle m \rangle| \lesssim 8.0 \times 10^{-3}$ eV, would lead to the conclusion that the neutrino mass spectrum is of the normal mass hierarchy type, $\Delta m_\odot^2 \equiv \Delta m_{21}^2$, and that m_1 is limited from above by $m_1 \leq (m_1)_{max} \simeq 5.8 \times 10^{-2}$ eV, where $(m_1)_{max}$ is determined by the condition $|\langle m \rangle|_{min} = |\langle m \rangle|_{exp}$, with $|\langle m \rangle|_{min}$ given by eq. (7). For the allowed values of the oscillation parameters (at a given confidence level, Fig. 3),

an upper bound on $|\langle m \rangle|$, $|\langle m \rangle| < 8 \times 10^{-4}$ eV, would imply an upper limit on m_1 , $m_1 < 0.01$ eV - Fig. 3, middle panel, and $m_1 < 0.025$ eV - Fig. 3, lower panel. For the best fit values of Δm_{atm}^2 , Δm_{\odot}^2 , θ_{\odot} and θ , the bound $|\langle m \rangle| \lesssim 8 \times 10^{-4}$ eV would lead to a rather narrow interval of possible values of m_1 , 1×10^{-3} eV $< m_1 < 4 \times 10^{-3}$ eV (Fig. 3, upper panel).

Thus, a measured value of (or an upper limit on) the effective Majorana mass $|\langle m \rangle| \lesssim 0.03$ eV would disfavor (if not rule out) the quasi-degenerate mass spectrum, while a value of $|\langle m \rangle| \lesssim 8 \times 10^{-3}$ eV would rule out the quasi-degenerate mass spectrum, disfavor the spectrum with inverted mass hierarchy and favor the hierarchical neutrino mass spectrum.

Using the best fit values of Δm_{\odot}^2 , $\cos 2\theta_{\odot}$ from [1] and of Δm_{atm}^2 and $\sin^2 \theta$ from [41], we have found that (Fig. 3, upper panel): i) $|\langle m \rangle| \lesssim 2.0 \times 10^{-3}$ eV in the case of neutrino mass spectrum with normal hierarchy, ii) 2.8×10^{-2} eV $\lesssim |\langle m \rangle| \lesssim 5.6 \times 10^{-2}$ eV if the spectrum is with inverted hierarchy, and iii) $|\langle m \rangle| \gtrsim 0.10$ eV for the quasi-degenerate mass spectrum. Therefore, if Δm_{atm}^2 , Δm_{\odot}^2 and $\cos 2\theta_{\odot}$ will be determined with a high precision ($\sim (10 - 15)\%$ uncertainty) using the data from the MINOS, KamLAND and the solar neutrino experiments and their best fit values will not change substantially with respect to those used in the present analysis ⁷, a measurement of $|\langle m \rangle| \gtrsim 0.03$ eV would rule out a hierarchical neutrino mass spectrum ($m_1 < m_2 \ll m_3$) even if there exists a factor of ~ 6 (or smaller) uncertainty in the value of $|\langle m \rangle|$ due to a poor knowledge of the corresponding nuclear matrix element(s). An experimental upper limit of $|\langle m \rangle| < 0.01$ eV suffering from the same factor of ~ 6 (or smaller) uncertainty would rule out the quasi-degenerate mass spectrum, while if the uncertainty under discussion is only by a factor which is not bigger than ~ 3.0 , the spectrum with inverted hierarchy would be strongly disfavored (if not ruled out).

If the minimal value of $\cos 2\theta_{\odot}$ inferred from the solar neutrino data, is somewhat smaller than that in eq. (2), the upper bound on $|\langle m \rangle|$ in the case of neutrino mass spectrum with normal hierarchy ($\Delta m_{\odot}^2 \equiv \Delta m_{21}^2$, $m_1 \ll 0.02$ eV) might turn out to be larger than the lower bound on $|\langle m \rangle|$ in the case of spectrum with inverted mass hierarchy ($\Delta m_{\odot}^2 \equiv \Delta m_{32}^2$, $m_1 \ll 0.02$ eV). Thus, there will be an overlap between the regions of allowed values of $|\langle m \rangle|$ in the two cases of neutrino mass spectrum at $m_1 \ll 0.02$ eV. The minimal value of $\cos 2\theta_{\odot}$ for which *the two regions do not overlap* is determined by the condition:

$$(\cos 2\theta_{\odot})_{\text{MIN}} = \frac{\sqrt{(\Delta m_{\odot}^2)_{\text{MAX}}} + 2\sqrt{(\Delta m_{\text{atm}}^2)_{\text{MAX}}} (\sin^2 \theta)_{\text{MAX}}}{2\sqrt{(\Delta m_{\text{atm}}^2)_{\text{MIN}}} + \sqrt{(\Delta m_{\odot}^2)_{\text{MAX}}}} + \mathcal{O}\left(\frac{(\Delta m_{\odot}^2)_{\text{MAX}}}{4(\Delta m_{\text{atm}}^2)_{\text{MIN}}}\right), \quad (20)$$

where we have neglected terms of order $(\sin^2 \theta)_{\text{MAX}}^2$. For the values of the neutrino oscillation parameters used in the present analysis this “border” value turns out to be $\cos 2\theta_{\odot} \cong 0.25$.

Let us note that [11] if the $(\beta\beta)_{0\nu}$ -decay is not observed, a measured value of $m_{\bar{\nu}_e}$ in ^3H β -decay experiments, $(m_{\bar{\nu}_e})_{\text{exp}} \gtrsim 0.35$ eV, which is larger than $(m_1)_{\text{max}}$, $(m_{\nu_e})_{\text{exp}} > (m_1)_{\text{max}}$, where $(m_1)_{\text{max}}$ is determined as in the *Case 1* (i.e., from the upper limit on $|\langle m \rangle|$, $|\langle m \rangle|_{\text{MIN}} = |\langle m \rangle|_{\text{exp}}$, with $|\langle m \rangle|_{\text{MIN}}$ given in eq. (7)), might imply that the massive neutrinos are Dirac particles. If the $(\beta\beta)_{0\nu}$ -decay has been observed and $|\langle m \rangle|$ measured, the inequality $(m_{\bar{\nu}_e})_{\text{exp}} > (m_1)_{\text{max}}$, would lead to the conclusion that there exist contribution(s) to the $(\beta\beta)_{0\nu}$ -decay rate other than due to the light Majorana neutrino exchange which partially cancel the contribution due to the Majorana neutrino exchange.

A measured value of $|\langle m \rangle|$, $(|\langle m \rangle|)_{\text{exp}} > 0.08$ eV, and a measured value of $m_{\bar{\nu}_e}$ or an upper bound on $m_{\bar{\nu}_e}$, such that $m_{\bar{\nu}_e} < (m_1)_{\text{min}}$, where $(m_1)_{\text{min}}$ is determined by the condition $|\langle m \rangle|_{\text{MAX}} = |\langle m \rangle|_{\text{exp}}$, with $|\langle m \rangle|_{\text{MAX}}$ given by eq. (14), would imply that [11] there are contributions to the $(\beta\beta)_{0\nu}$ -decay rate in addition to the ones due to the light Majorana neutrino

⁷The conclusions that follow practically do not depend on $\sin^2 \theta < 0.05$.

exchange (see, e.g., [45]), which enhance the $(\beta\beta)_{0\nu}$ -decay rate. This would signal the existence of new $\Delta L = 2$ processes beyond those induced by the light Majorana neutrino exchange in the case of left-handed charged current weak interaction.

4 Conclusions

Assuming 3- ν mixing and massive Majorana neutrinos, we have analyzed the implications of the results of the solar neutrino experiments, including the latest SNO data, which favor the LMA MSW solution of the solar neutrino problem with $\tan^2 \theta_\odot < 1$, for the predictions of the effective Majorana mass $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$ -decay. Neutrino mass spectra with normal mass hierarchy, with inverted hierarchy and of quasi-degenerate type are considered. For $\cos 2\theta_\odot \geq 0.26$, which follows (at 99.73% C.L.) from the analysis of the solar neutrino data performed in [1], we find significant lower limits on $|\langle m \rangle|$ in the cases of quasi-degenerate and inverted hierarchy neutrino mass spectrum, $|\langle m \rangle| \gtrsim 0.03$ eV and $|\langle m \rangle| \gtrsim 8.5 \times 10^{-3}$ eV, respectively. If the neutrino mass spectrum is hierarchical (with inverted hierarchy), the upper limit holds $|\langle m \rangle| \lesssim 8.2 \times 10^{-3}$ (8.0×10^{-2}) eV. Correspondingly, not only a measured value of $|\langle m \rangle| \neq 0$, but even an experimental upper limit on $|\langle m \rangle|$ of the order of few $\times 10^{-2}$ eV can provide information on the type of the neutrino mass spectrum; it can provide also a significant upper limit on the mass of the lightest neutrino m_1 . Further reduction of the LMA solution region due to data, e.g., from the experiments SNO, KamLAND and BOREXINO, leading, in particular, to an increase (a decreasing) of the current lower (upper) bound of $\cos 2\theta_\odot$ can strengthen further the above conclusions.

Using the best fit values of Δm_\odot^2 , $\cos 2\theta_\odot$ from [1] and of Δm_{atm}^2 and $\sin^2 \theta$ from [41], we have found that (Fig. 3, upper panel): i) $|\langle m \rangle| \lesssim 2.0 \times 10^{-3}$ eV in the case of neutrino mass spectrum with normal hierarchy, ii) 2.8×10^{-2} eV $\lesssim |\langle m \rangle| \lesssim 5.6 \times 10^{-2}$ eV if the spectrum is with inverted hierarchy, and iii) $|\langle m \rangle| \gtrsim 0.10$ eV for the quasi-degenerate neutrino mass spectrum. Therefore, if Δm_{atm}^2 , Δm_\odot^2 and $\cos 2\theta_\odot$ will be determined with a high precision ($\sim (10 - 15)\%$ uncertainty) using the data from the MINOS, KamLAND and the solar neutrino experiments and their best fit values will not change substantially with respect to those used in the present analysis, a measurement of $|\langle m \rangle| \gtrsim 0.03$ eV would rule out a hierarchical neutrino mass spectrum ($m_1 < m_2 \ll m_3$) even if there exists a factor of ~ 6 uncertainty in the value of $|\langle m \rangle|$ due to a poor knowledge of the corresponding nuclear matrix element(s). An experimental upper limit of $|\langle m \rangle| < 0.01$ eV suffering from the same factor of ~ 6 (or smaller) uncertainty would rule out the quasi-degenerate neutrino mass spectrum, while if the uncertainty under discussion is by a factor not bigger than ~ 3.0 , the spectrum with inverted hierarchy would be strongly disfavored (if not ruled out).

Finally, a measured value of $|\langle m \rangle| \gtrsim 0.2$ eV, which would imply a quasi-degenerate neutrino mass spectrum, combined with data on neutrino masses from the ^3H β -decay experiment KATRIN (an upper limit or a measured value ⁸), might allow to establish whether the CP-symmetry is violated in the lepton sector.

Note Added.

After the work on the present study was essentially completed, few new global analyses of the solar neutrino data have appeared [47, 48, 49, 50]. The results obtained in [47] do not differ substantially from those derived in [1]; in particular, the (99.73% C.L.) minimal allowed values of $\cos 2\theta_\odot$ in the LMA solution region found in [1] and in [47] practically coincide. The best fit values of Δm_\odot^2 and $\cos 2\theta_\odot$ found in [1, 47, 49, 50] also practically coincide, with $\cos 2\theta_\odot|_{\text{BF}}$ lying in the interval (0.41 - 0.50) and $\Delta m_\odot^2|_{\text{BF}} \simeq 5 \times 10^{-5}$ eV². The authors of [48] find a similar $\cos 2\theta_\odot|_{\text{BF}}$, but a somewhat larger $\Delta m_\odot^2|_{\text{BF}} \simeq 7.9 \times 10^{-5}$ eV². According to [48], [49] and [50], the lower limit $\cos 2\theta_\odot > 0.25$ holds approximately at 94% C.L., 90% C.L. and 81% C.L., respectively. Larger

⁸ Information on the absolute values of neutrino masses in the range of interest might be obtained also from cosmological and astrophysical data, see, e.g., ref. [46].

maximal allowed values of Δm_{\odot}^2 than that given in eq. (1) - of the order of $(4 - 5) \times 10^{-4} \text{ eV}^2$ (99.73% C.L.), have been obtained in the analyses performed in [48, 49, 50]. The authors of [1, 47] used the full SNO data on the day and night event spectra [1] in their analyses, while the authors of [48, 49, 50] did not use at all or used only part of these data.

References

- [1] SNO Collaboration, Q.R. Ahmad et al., nucl-ex/0204009.
- [2] SNO Collaboration, Q.R. Ahmad et al., nucl-ex/0204008.
- [3] SNO Collaboration, Q.R. Ahmad et al., *Phys. Rev. Lett.* **87** (2001) 071301 (nucl-ex/0106015).
- [4] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, *Phys. Rev. Lett.* **86** (2001) 5651 and 5656.
- [5] B.T. Cleveland et al., *Astrophys. J.* **496** (1998) 505; Y. Fukuda *et al.*, *Phys. Rev. Lett.* **77** (1996) 1683; V. Gavrin, *Nucl. Phys. Proc. Suppl.* **91** (2001) 36; W. Hampel *et al.*, *Phys. Lett.* **B447** (1999) 127; M. Altmann *et al.*, *Phys. Lett.* **B490** (2000) 16.
- [6] Super-Kamiokande Collaboration, H. Sobel et al., *Nucl. Phys. Proc. Suppl.* **91** (2001) 127.
- [7] S. M. Bilenky, C. Giunti and W. Grimus, *Prog. Part. Nucl. Phys.* **43** (1999) 1.
- [8] S.T. Petcov, hep-ph/9907216.
- [9] S.M. Bilenky and S.T. Petcov, *Rev. Mod. Phys.* **59** (1987) 67.
- [10] S.M. Bilenky, S. Pascoli and S.T. Petcov, *Phys. Rev.* **D64** (2001) 053010.
- [11] S. Pascoli, S.T. Petcov and L. Wolfenstein, *Phys. Lett.* **B524** (2002) 319; S. Pascoli and S.T. Petcov, hep-ph/0111203.
- [12] B. Pontecorvo, *Zh. Eksp. Teor. Fiz.* **33**, 549 (1957), and **34**, 247 (1958).
- [13] Z. Maki, M. Nakagawa and S. Sakata, *Prog. Theor. Phys.* **28** (1962) 870.
- [14] S. M. Bilenky *et al.*, *Phys. Lett.* **B94** (1980) 495.
- [15] M. Doi *et al.*, *Phys. Lett.* **B102** (1981) 323.
- [16] L. Wolfenstein, *Phys. Lett.* **B107** (1981) 77.
- [17] S. M. Bilenky, N. P. Nedelcheva and S. T. Petcov, *Nucl. Phys.* **B247** (1984) 589; B. Kayser, *Phys. Rev.* **D30** (1984) 1023.
- [18] H.V. Klapdor-Kleingrothaus *et al.*, *Mod. Phys. Lett.* **16** (2001) 2409.
- [19] F. Feruglio, A. Strumia and F. Vissani, hep-ph/0201291.
- [20] C.E. Aalseth *et al.*, hep-ex/0202018.
- [21] S. T. Petcov and A. Yu. Smirnov, *Phys. Lett.* **B322** (1994) 109.
- [22] S.M. Bilenky *et al.*, *Phys. Rev.* **D54** (1996) 4432.
- [23] S.M. Bilenky *et al.*, *Phys. Lett.* **B465** (1999) 193.
- [24] V. Barger and K. Whisnant, *Phys. Lett.* **B456** (1999) 194; H. Minakata and O. Yasuda, *Nucl. Phys.* **B523** (1998) 597; T. Fukuyama *et al.*, *Phys. Rev.* **D57** (1998) 5844 and hep-ph/0204254; P. Osland and G. Vigdel, *Phys. Lett.* **B520** (2001) 128; D. Falcone and F. Tramontano, *Phys. Rev.* **D64** (2001) 077302; T. Hambye, hep-ph/0201307.
- [25] S.M. Bilenky, S. Pascoli and S.T. Petcov, *Phys. Rev.* **D64** (2001) 113003.

- [26] F. Vissani, *JHEP* **06** (1999) 022; M. Czakon *et al.*, *Phys. Lett.* **B465** (1999) 211, hep-ph/0003161 and *Phys. Rev.* **D65** (2002) 053008; H. V. Klapdor-Kleingrothaus, H. Pas and A. Yu. Smirnov, *Phys. Rev.* **D63** (2001) 073005; H. Minakata and H. Sugiyama, *Phys. Lett.* **B526** (2002) 335; Z. Xing, *Phys. Rev.* **D65** (2002) 077302; N. Haba, N. Nakamura and T. Suzuki, hep-ph/0205141.
- [27] A. Osipowicz *et al.* (KATRIN Project), hep-ex/0109033.
- [28] W. Rodejohann, *Nucl. Phys.* **B597** (2001) 110, and hep-ph/0203214.
- [29] H. V. Klapdor-Kleingrothaus *et al.*, *Nucl. Phys. Proc. Suppl.* **100** (2001) 309.
- [30] C.E. Aalseth, F.T. Avignone III *et al.*, *Physics of Atomic Nuclei* **63** (2000) 1225.
- [31] X. Sarazin *et al.* (NEMO3 Collaboration), hep-ex/0006031.
- [32] E. Fiorini, *Phys. Rep.* **307** (1998) 309.
- [33] H. V. Klapdor-Kleingrothaus *et al.*, *J. Phys. G* **24** (1998) 483.
- [34] M. Danilov *et al.*, *Phys. Lett.* **B480** (2000) 12.
- [35] L. De Braekeleer *et al.* (Majorana Project), *Proceedings of the Carolina Conference on Neutrino Physics*, Columbia (SC), USA, March 2000.
- [36] H. Ejiri *et al.*, *Phys. Rev. Lett.* **85** (2000) 2917.
- [37] V. Lobashev *et al.*, *Nucl. Phys. Proc. Suppl.* **91** (2001) 280.
- [38] Ch. Weinheimer *et al.*, talk at the Int. Conf. on Neutrino Physics and Astrophysics “Neutrino’02”, May 25 - 30, 2002, Munich, Germany.
- [39] M. Appolonio *et al.*, *Phys. Lett.* **B466** (1999) 415.
- [40] F. Boehm *et al.*, *Phys. Rev. Lett.* **84** (2000) 3764 and *Phys. Rev.* **D62** (2000) 072002.
- [41] C. Gonzalez-Garcia *et al.*, *Phys. Rev.* **D63** (2001) 033005.
- [42] D. Michael (MINOS Collaboration), talk at the Int. Conf. on Neutrino Physics and Astrophysics “Neutrino’02”, May 25 - 30, 2002, Munich, Germany.
- [43] J. Shirai (KamLAND Collaboration), talk at the Int. Conf. on Neutrino Physics and Astrophysics “Neutrino’02”, May 25 - 30, 2002, Munich, Germany.
- [44] A. de Gouvêa and C. Peña-Garay, hep-ph/0107186.
- [45] K.S. Babu and R.N. Mohapatra, *Phys. Rev. Lett.* **75** (1995) 2276; H. Pas *et al.*, *Phys. Lett.* **B398** (1997) 311; *ibid.* **B459** (1999) 450.
- [46] T. J. Weiler, *Astropart. Phys.* **11** (1999) 303; H. Pas and T. J. Weiler, *Phys. Rev.* **D63** (2001) 113015; Z. Fodor *et al.*, *Phys. Rev. Lett.* **88** (2002) 171101.
- [47] V. Barger *et al.*, hep-ph/0204253, version 2 of 29 of April, 2002.
- [48] P. Creminelli, G. Signorelli and A. Strumia, hep-ph/0102234, version 3 of 22 of April, 2002.
- [49] A. Bandyopadhyay *et al.*, hep-ph/0204286.
- [50] J.N. Bachall, M.C. Gonzalez-Garcia and C. Peña-Garay, hep-ph/0204314.

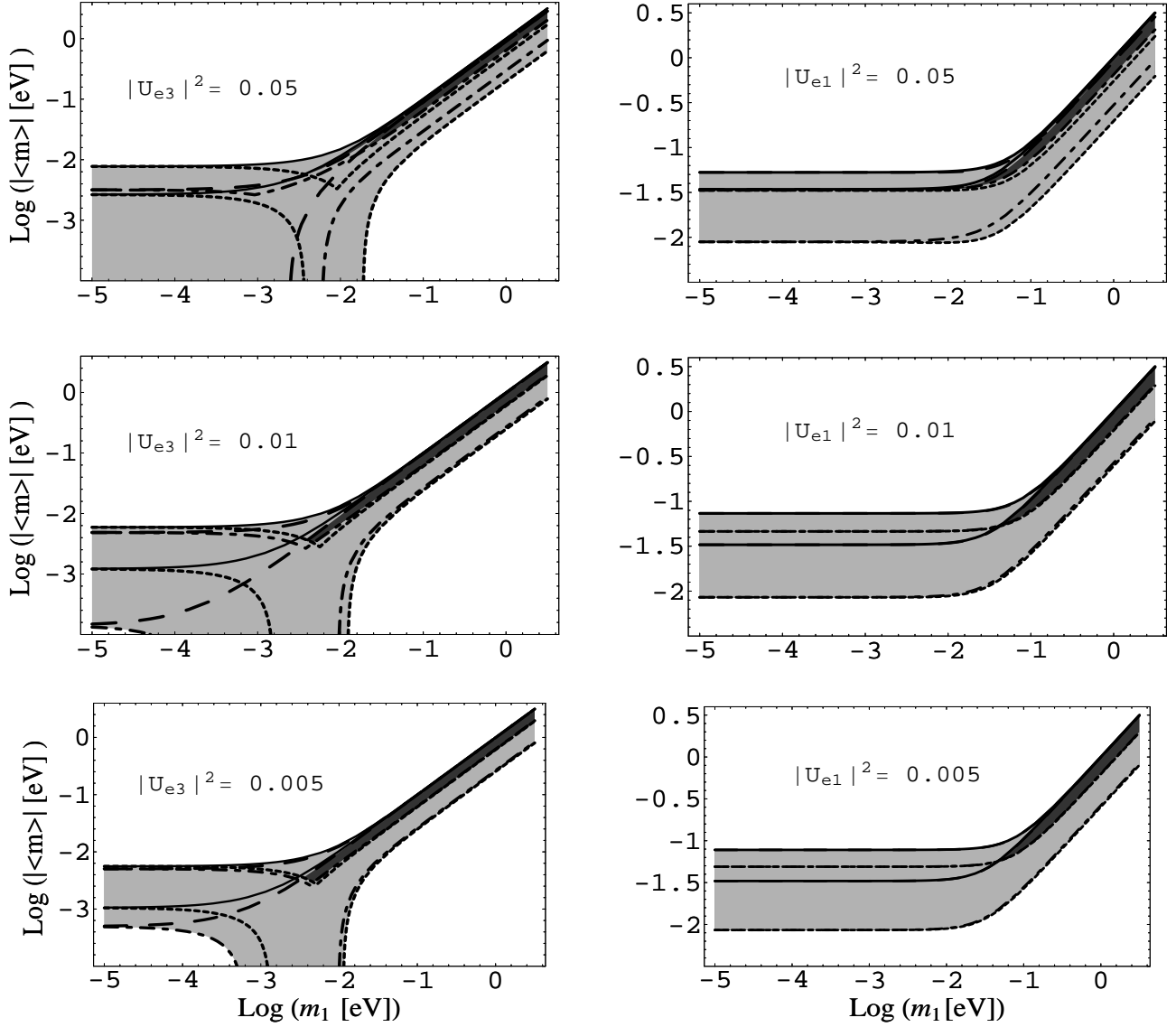


Figure 1: The dependence of $|\langle m \rangle|$ on m_1 in the case of the LMA solution of the solar neutrino problem [1] (99.73% C.L.), for i) $\Delta m_{\odot}^2 = \Delta m_{21}^2$ (left panels) and ii) $\Delta m_{\odot}^2 = \Delta m_{32}^2$ (right panels) and for $\sin^2 \theta = 0.05$ (upper panels), $\sin^2 \theta = 0.01$ (middle panels), $\sin^2 \theta = 0.005$ (lower panels). For $\Delta m_{\odot}^2 = \Delta m_{21}^2$, ($\sin^2 \theta = |U_{e3}|^2$), the allowed values of $|\langle m \rangle|$ are constrained to lie in the case of CP-conservation in the medium-grey regions a) between the two thick solid lines if $\eta_{21} = \eta_{31} = 1$, b) between the two long-dashed lines and the axes if $\eta_{21} = -\eta_{31} = 1$, c) between the dash-dotted lines and the axes if $\eta_{21} = -\eta_{31} = -1$, d) between the short-dashed lines if $\eta_{21} = \eta_{31} = -1$. For $\Delta m_{\odot}^2 = \Delta m_{32}^2$, ($\sin^2 \theta = |U_{e1}|^2$), the allowed regions for $|\langle m \rangle|$ correspond: for $|U_{e1}|^2 = 0.005$ and $|U_{e1}|^2 = 0.01$ - to the medium-grey regions a) between the solid lines if $\eta_{21} = \eta_{31} = \pm 1$, b) between the dashed lines if $\eta_{21} = -\eta_{31} = \pm 1$, and for $|U_{e1}|^2 = 0.05$ - to the medium-grey regions c) between the solid lines if $\eta_{21} = \eta_{31} = 1$, d) between the long-dashed lines if $\eta_{21} = \eta_{31} = -1$, e) between the dashed-dotted lines if $\eta_{21} = -\eta_{31} = 1$, f) between the short-dashed lines if $\eta_{21} = -\eta_{31} = -1$. In the case of CP-violation, the allowed region for $|\langle m \rangle|$ covers all the grey regions. Values of $|\langle m \rangle|$ in the dark grey regions signal CP-violation.

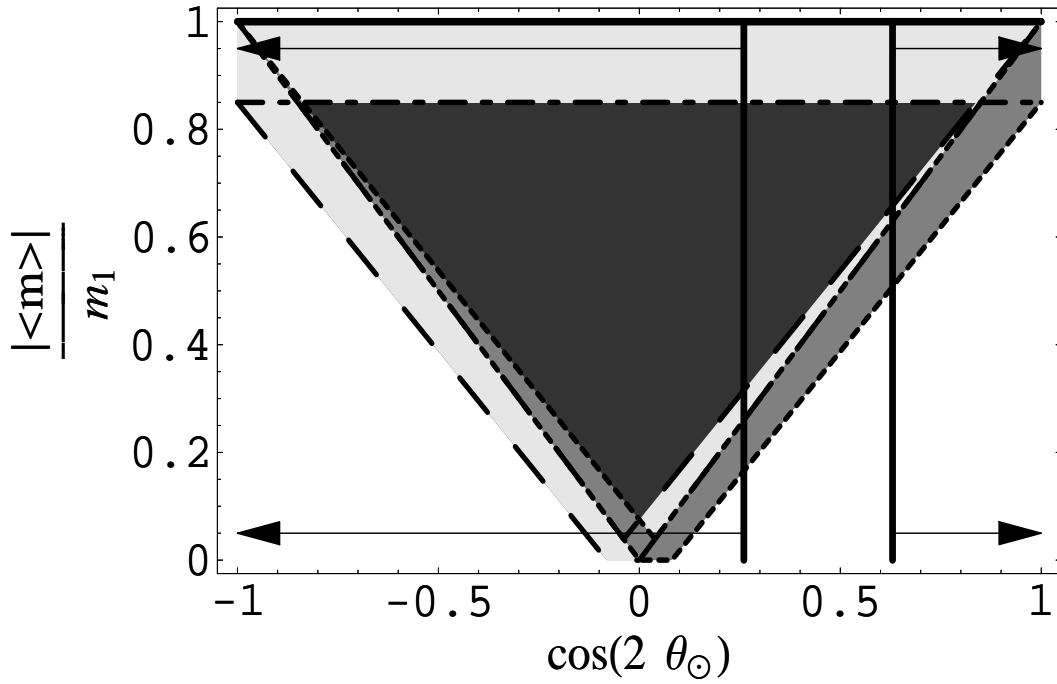


Figure 2: The dependence of $|\langle m \rangle|/m_1$ on $\cos 2\theta_\odot$ for the quasi-degenerate neutrino mass spectrum ($m_1 > 0.2$ eV, $m_1 \simeq m_2 \simeq m_3 \simeq m_{\bar{\nu}_e}$). If CP-invariance holds, the values of $|\langle m \rangle|/m_1$ lie: i) for $\eta_{21} = \eta_{31} = 1$ - on the line $|\langle m \rangle|/m_1 = 1$, ii) for $\eta_{21} = -\eta_{31} = 1$ - in the region between the thick horizontal solid and dash-dotted lines (in light grey and medium grey colors), iii) for $\eta_{21} = -\eta_{31} = -1$ - in the light grey polygon with long-dashed and long-dashed-double-dotted line contours and iv) for $\eta_{21} = \eta_{31} = -1$ - in the medium grey polygon with the short-dashed and long-dashed-double-dotted line contours. The “just-CP-violation” region is denoted by dark-grey color. The values of $\cos 2\theta_\odot$ between the doubly thick solid lines correspond to the lower and upper limits of the LMA solution regions found in ref. [1] at 99.73% C.L.

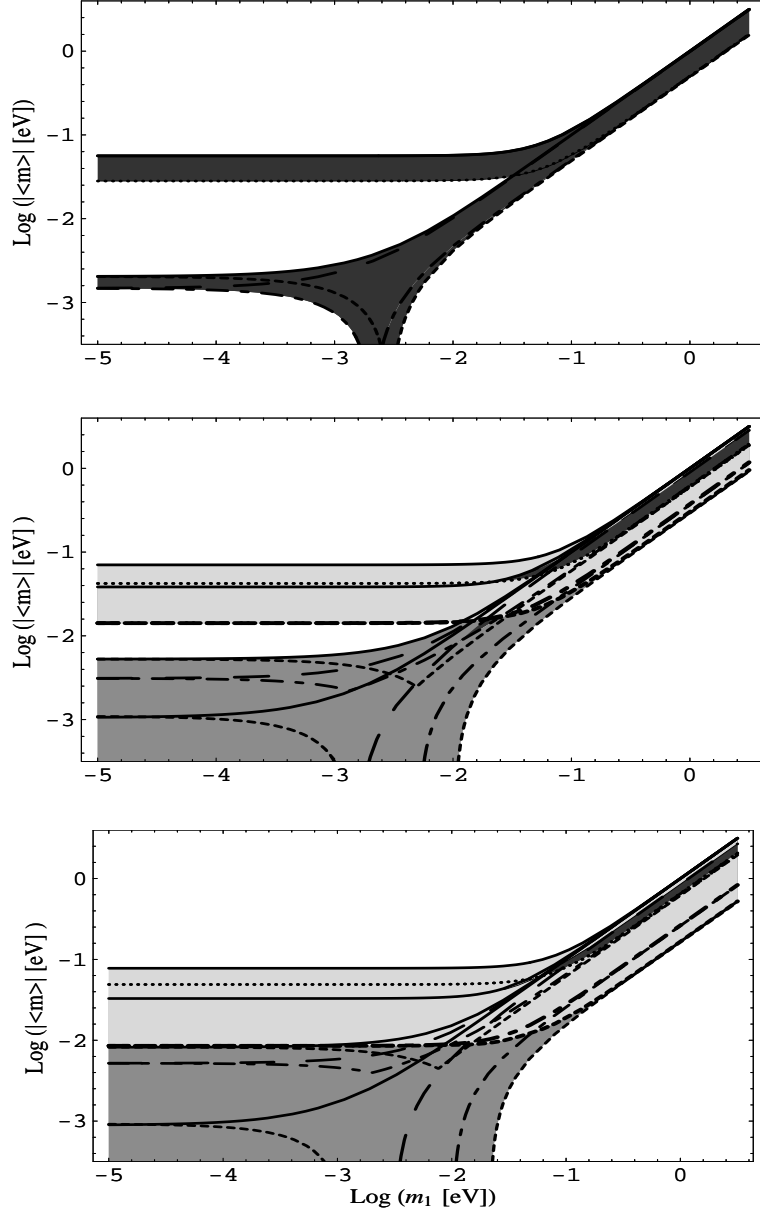


Figure 3: The dependence of $|\langle m \rangle|$ on m_1 in the case of the LMA solution, for $\Delta m_{\odot}^2 = \Delta m_{21}^2$ and $\Delta m_{\odot}^2 = \Delta m_{32}^2$, and for the best fit values (upper panel) and the 90% C.L. allowed values (middle panel) of the neutrino oscillation parameters found in refs. [1, 41]. The lower panel is obtained by using the 99.73% C.L. allowed values of Δm_{\odot}^2 and $\cos 2\theta_{\odot}$ from [1] and the 99% C.L. allowed values of Δm_{atm}^2 and $\sin^2 \theta$ from [41] (the latter article does not include results at 99.73% C.L.). In the case of CP-conservation, the allowed values of $|\langle m \rangle|$ are constrained to lie: for i) $\Delta m_{\odot}^2 = \Delta m_{21}^2$ and the middle and lower panels (upper panel) - in the medium-grey and light-grey regions a) between the two lower thick solid lines (on the lower thick solid line) if $\eta_{21} = \eta_{31} = 1$, b) between the two long-dashed lines and the axes (on the long-dashed line) if $\eta_{21} = -\eta_{31} = 1$, c) between the two thick dash-dotted lines and the axes (on the dash-dotted lines) if $\eta_{21} = -\eta_{31} = -1$, d) between the three thick short-dashed lines and the axes (on the short-dashed lines) if $\eta_{21} = \eta_{31} = -1$; and for ii) $\Delta m_{\odot}^2 = \Delta m_{32}^2$ and the middle and lower panels (upper panel) - in the light-grey regions a) between the two upper thick solid lines (on the upper thick solid line) if $\eta_{21} = \eta_{31} = \pm 1$, b) between the dotted and the doubly-thick short-dashed lines (on the dotted line) if $\eta_{21} = -\eta_{31} = -1$, c) between the dotted and the doubly-thick dash-dotted lines (on the dotted line) if $\eta_{21} = -\eta_{31} = +1$. In the case of CP-violation, the allowed regions for $|\langle m \rangle|$ cover all the grey regions. Values of $|\langle m \rangle|$ in the dark grey regions signal CP-violation.